Geometric Analysis in Frauenwörth

Book of Abstracts

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About

After a long and tough winter overshadowed by the current pandemic, let us use the summer to meet in person again and do some math!

This workshop brings together experts in geometric analysis, calculus of variations, numerics, and optimization on the island Fraueninsel right in the middle of the Chiemsee. It will be a small workshop as we used to have them before the CoVid-19 crisis: A list of interesting and inspiring talks and lots of time for discussions and collaborations. But most importantly: No Zoom or other online tools but in person!

Organizers

Simon Blatt (University Salzburg)
Philipp Reiter (University Chemnitz)
Armin Schikorra (University Pittsburgh)

Timetable

Monday, 16th of August

8:00–9:00	**Breakfast**	
9:00–10:00	**Julian Scheuer** University Cardiff, United Kingdom	Stability from rigidity via umbilicity
10:00–10:30	**Coffee**	
10:30–11:30	**Remy Rodiac** University Paris-Saclay	Towards a relaxed energy for the neo-Hookean energy in the axially symmetric setting
12:00–15:00	**Lunch**	
15:00-16:00	**Claus Ernst** Western Kentucky University, USA	Knot Diagrams without inflection points.
16:00–16:30	**Coffee**	
16:30-17:30	**Daniel Steenebrügge** RWTH Aachen, Germany	Analyticity of Critical Points of Generalized Integral Menger Curvature
18:00	**Dinner**	

Tuesday, 17 of August

8:00–9:00	**Breakfast**	
9:00 –10:00	**Jonas Hirsch** University Leipzig, Germany	Area minimizing hypersurfaces mod(p): A geometric free boundary problem
10:00–10:30	**Coffee**	
10:30–11:30	**Henrik Schumacher** University Chemnitz, Germany	Repulsive Curves and Surfaces
12:00–14:30	**Lunch**	
14:30–15:30	**Nicole Vorderobermeier** University Salzburg, Austria	On the gradient flow of the p-elastic energy for curves
15:30–16:00	**Coffee**	
16:00-17:00	**Rhoslyn Coles** University Potsdam, Germany	Experiments with Tubes: Curvature measures and the thermodynamics of solvation.
17:00-18:00	**Stefan Krömer** Czech Academy of Sciences, Czech republic	Relaxation of functionals with linear growth: Interaction of emerging measures and free discontinuities
18:00	**Dinner**	

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8:00 - 9:00	**Breakfast**
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8:00–9:00	**Breakfast**	
9:00 –10:00	**Heiko von der Mosel** RWTH Aachen, Germany	The Calugareanu-Fuller-White Theorem revisted
10:00–10:30	**Coffee**	
10:30–11:30	**Sören Bartels** University Freiburg, Germany	Simulation of Free Boundary Problems in the Nonlinear Bending of Elastic Rods and Plates
12:00–14:30	**Lunch**	
14:30–15:30	**Ernst Kuwert** University Freiburg, Germany	Curvature varifolds with orthogonal boundary
15:30–16:00	**Coffee**	
16:00-17:00	**Andreas Gastel** University Duisburg/ Essen, Germany	Harmonic maps and Cosserat elasticity
17:00-18:00	**Katarzyna Mazowieck** RWTH Aachen, Germany	Quantitative characterization of traces of Sobolev maps
18:00	**Dinner**	

Friday, 20th of August

8:00–9:00	**Breakfast**	
9:00 –10:00	**Guofang Wang** University Freiburg, Germany	A new Sobolev type inequality
10:00–10:30	**Coffee**	
10:30–11:30	**Tobias Lamm** KIT Karlsruhe, Germany	Diffusive stability results for the harmonic map flow and related equations
12:00	**Lunch and Departure**	

List of Abstracts

Monday 16th

Approximation of Fractional Operators in Harmonic Maps, Imaging, and Geophysics

Harbir Antil

George Manson University, USA

Fractional calculus and its application to anomalous diffusion has recently received a tremendous amount of attention. In complex/heterogeneous material mediums, the long-range correlations or hereditary material properties are presumed to be the cause of such anomalous behavior. Owing to the revival of fractional calculus, these effects are now conveniently modeled by fractional-order differential operators and the governing equations are reformulated accordingly.

This talk will address approximation of fractional harmonic maps, fractional regularization in imaging and fractional Helmholtz equations arising in geophysics. Many challenges arise, for instance, in harmonic maps, besides the unit-length constraint, one has to tackle the difficulty of nonlocality. In imaging, even though the new regularization is differentiable, but the analysis needs to be carried out in weighted Sobolev spaces. If time permits, optimal control of fractional PDEs will be discussed as well.

Stability from rigidity via umbilicity

Julian Scheuer

University Cardiff, United Kingdom

The soap bubble theorem says that a closed, embedded surface of the Euclidean space with constant mean curvature must be a round sphere. Especially in real-life problems it is of importance whether and to what extent this phenomenon is stable, i.e. when a surface with almost constant mean curvature is close to a sphere. This problem has been receiving lots of attention until today, with satisfactory recent solutions due to Magnanini/Pogessi and Ciraolo/Vezzoni.

The purpose of this talk is to discuss further problems of this type and to provide two approaches to their solutions. The first one is a new general approach based on stability of the so-called "Nabelpunktsatz". The second one is of variational nature and employs the theory of curvature flows.

Quantitative characterization of traces of Sobolev maps

Katarzyna Mazowieck

RWTH Aachen, Germany

We give a quantitative characterization of traces on the boundary of Sobolev maps in $\dot{W}^{1,p}(\mathcal{M},\mathcal{N})$, where $\mathcal{M}$ and $\mathcal{N}$ are compact Riemannian manifolds, $\partial\mathcal{M} \neq \emptyset$: the Borel-measurable maps $u\colon \partial\mathcal{M} \to \mathcal{N}$ that are the trace of a map $U \in \dot{W}^{1,p}(\mathcal{M},\mathcal{N})$ are characterized as the maps for which there exists an extension energy density $w\colon \partial\mathcal{M} \to [0,\infty]$ that controls the Sobolev energy of extensions from $\lfloor p-1 \rfloor$-dimensional subsets of $\partial\mathcal{M}$ to $\lfloor p \rfloor$-dimensional subsets of $\mathcal{M}$.

Joint work with Jean Van Schaftingen.

Knot Diagrams without inflection points.

Claus Ernst

George Mason University, USA

There exist several indices of knots (and links) that depend on what type of diagrams of that knot (or link) exist. The bridge index ($br(K)$) indicates the number of maxima that are needed in any diagram of the knot, the braid index ($b(K)$) indicates the smallest number of times the strand must circle around an axis in a knot diagram (called a braid diagram), the spiral index ($sp(K)$) indicates how many "spirals" are needed in a diagram without inflection points. We have $br(K) \leq sp(K) \leq b(K)$. We call a knot a bb-knot if $br(K) = b(K)$ and we call a knot curly if $sp(K) < b(K)$. Several example of bb-knots and curly knots will be given.

Analyticity of Critical Points of Generalized Integral Menger Curvature

Daniel Steenebrügge

RWTH Aachen, Germany

The following is based on joint work with Nicole Vorderobermeier, Universität Salzburg. Generalized Integral Menger Curvature is a knot energy, i.e. a functional on the set of closed, embedded curves that punishes self-intersections. In addition to being good representatives of their knot classes (configurations that can be achieved without cutting and glueing), local minimizers of such knot energies have long been expected to have especially nice properties. In the case of a subfamily of Generalized Integral Menger Curvature, we show that this is indeed true: Critical points are not only smooth, as was shown before by Simon Blatt and Philipp Reiter, but analytic. Our approach is inspired by analyticity results on critical points for O'Hara's knot energies by Blatt and Vorderobermeier which are in turn based on Cauchy's method of majorants and a decomposition of the first variation. The main new idea is an additional iteration in the recursive estimate of the derivatives to obtain a sufficient difference in the order of regularity.

Tuesday 17th

Area minimizing hypersurfaces mod(p): A geometric free boundary problem

Jonas Hirsch

University Leipzig, Germany

joint work with C. De Lellis, A Marches, S. Stuvard and L. Spolaor

In this talk I would like to give an idea of our resent result on the structure of area minimizing hypersurfaces mod(p).

Motivation: If one considers real soap films one notice that from time to time one can find configurations where different soap films join on a common piece. One possibility to allow this kind of phenomenon is to consider flat chains with coefficients in $\mathbb{Z}_p$. For instance for $p = 2$ one can deal with unoriented surfaces, for $p = 3$ one allows triple junctions. Using known results it can be shown that for $p = 3$ this common piece is itself nicely regular. It was our aim to investigate the situation for higher p.

We consider area minimizing m-dimensional currents mod(p) in complete C^2 Riemannian manifolds Σ of dimension $m + 1$. For odd moduli we prove that, away from a closed rectifiable set of codimension 2, the current in question is, locally, the union of finitely many smooth minimal hypersurfaces coming together at a common $C^{1,\alpha}$ boundary of dimension $m - 1$, and the result is optimal. For even p such structure holds in a neighborhood of any point where at least one tangent cone has $(m - 1)$-dimensional spine. These structural results are indeed the byproduct of a theorem that proves (for any modulus) uniqueness and decay towards such tangent cones. The underlying strategy of the proof is inspired by the techniques developed by Simon in a class of *multiplicity one* stationary varifolds. The major difficulty in our setting is produced by the fact that the cones and surfaces under investigation have arbitrary multiplicities ranging from 1 to $\lfloor \frac{p}{2} \rfloor$.

Repulsive Curves and Surfaces

Henrik Schumacher

University Chemnitz, Germany

I am going to report on recent work on the numerical optimization of tangent-point energies of curves and surfaces. After a motivation and brief introduction to the central computational tools — construction of suitable Riemannian metrics on the space of embedded manifolds, a polyhedral discretization of the energies, and fast multipole techniques — there will be plenty of opportunity to admire the shear beauty of the energies' minimizers and gradient flows.

This is based on joint work with Philipp Reiter (Technical University Chemnitz) and Caleb Brakensiek, Keenan Crane, and Chris Yu (Carnegie Mellon University, Pittsburgh).

On the gradient flow of the p-elastic energy for curves

Nicole Vorderobermeier

University Salzburg

Motivated from applications to geometric knot theory, we investigate the negative L^2-gradient flow of the p-elastic energy for $p > 2$, with an additive positive multiple of the length of the curve. While the L^2-gradient flow of the p-elastic energy has been extensively studied when $p = 2$, relatively little is known in the $p > 2$ case, which exhibits an additional degeneracy. We tackle these degenerate evolution equations via two different techniques, namely de Giorgi's minimizing movement scheme and the vanishing viscosity method. After explaining these approaches, we present the respective results on the existence and convergence of solutions and state related open problems. The talk is based on joint work with Simon Blatt and Christopher Hopper.

Experiments with Tubes: Curvature measures and the thermodynamics of solvation

Rhoslyn Coles

University Potsdam, Germany

Our motivating problem is the dynamics of long polyatomic molecules, such as proteins, in solution. The interaction of the fluid and protein, a process which strives to decrease the thermodynamic potential energy of the system, is decisive for the protein's functionality. Using a very simple tube model, we explore the importance of geometry in this interaction.

Such a study is possible because of the coupling between thermodynamics and shape: The energy of the system may be evaluated via the curvature measures of an abstract geometry associated to the system. Here, this geometry is simply the tubular neighbourhood of a certain radius surrounding a curve. In this talk I will introduce this problem and the computational tool I have developed to explore low energy curve configurations.

This work is part of my doctoral studies under the guidance of Prof. Myf Evans (Univ. Potsdam)

Relaxation of functionals with linear growth: Interaction of emerging measures and free discontinuities

Stefan Krömer

Czech Academy of Sciences, Czeck Republic

For an integral functional defined on functions (u, v) in $W^{1,1} \times L^1$ featuring a strong interaction term between u and v, we calculate its relaxation in the space of functions with bounded variations and Radon measures. In particular, this requires a precise understanding of the behavior of minimizing sequences. As it turns out, these can exhibit interesting oscillatory and concentrating behavior even if the leading terms in the functional (in the sense of highest order derivatives) are convex and coercive.

Joint work: Martin Kruĭík (Prague), Elvira Zappale (Rome)

Thursday 19th

The Calugareanu-Fuller-White Theorem revisted

Heiko von der Mosel

University Aachen, Germany

The Calugareanu-Fuller-White Theorem unites the Link, the Twist and the Writhe of a smooth closed ribbon with the formula

$$Lk = Tw + Wr.$$

Motivated by issues that arise when the ribbon is non-closed we report on some preliminary results regarding lower regularity of the base curve.

This is joint work with John H. Maddocks.

Simulation of Free Boundary Problems in the Nonlinear Bending of Elastic Rods and Plates

Sören Bartels

University Freiburg, Germany

The mathematical description of large bending deformations of thin elastic rods and plates leads to fourth order problems with nonlinear pointwise constraints that give rise to various free boundary phenomena. The free boundary may separate regions of trivial and large deformations, describe the contact zone in the presence of an obstacle, or be related to the occurrence of self-contact. We devise and analyse numerical methods that are capable of reliably capturing these effects under minimal regularity assumptions and which allow us to experimentally study topological transitions of free boundaries.

Curvature varifolds with orthogonal boundary

Ernst Kuwert

University Freiburg, Germany

For a bounded domain $\Omega \subset \mathbb{R}^n$, we consider the class of m-dimensional surfaces $\Sigma \subset \overline{\Omega}$ which meet $\partial\Omega$ orthogonally along the boundary. We study the problem of estimating the area of Σ in terms of the L^p integral of its curvature.

Joint work with Marius Müller, Freiburg

Harmonic maps and Cosserat elasticity

Andreas Gastel

University Duisburg / Essen

In some nonlinear models for micropolar elasticity, a harmonic map type equation is coupled with other equations. Hence, in order to understand such models, we would like to learn how far harmonic map theory may help. But even using that, basic questions about the existence and regularity of weak solutions remain open. We discuss similarities and differences between Cosserat elasticity and harmonic maps.

Towards a relaxed energy for the neo-Hookean energy in the axially symmetric setting

Remy Rodiac

University Paris-Saclay

We consider the problem of minimizing hyperelastic energies in 3D. It is known that if the energy space allows for maps with cavitation (creation of holes) then the hyperelastic energies are not lower semi-continuous for the weak convergence . We can then try to find minimizers in a subset of maps not allowing for cavitation, but for the neo-Hookean energy this condition is not closed under the weak convergence in H^1. I will describe some properties of weak limits of minimizng sequences and give a lower-bound on the relaxed energy for the neo-Hookean problem. This problem bears some analogy with the problem of finding smooth harmonic maps from B^3 into S^2 with a smooth prescribed data of degree 0 on a boundary.

This is a joint work with Marco Barchiesi, Duvan Henao and Carlos Mora-Corral.

Friday 20th

A new Sobolev type inequality

Guofang Wang

Unversity Freiburg, Germany

We will talk about a "new" Sobolev type inequality, which was in fact obtained joint with Pengfei Guan in 2005. We will talk about its background, its application in the proof of the Alexandrov-Fenchel inequality in the hyperbolic space and its potential generalizations.

Diffusive stability results for the harmonic map flow and related equations

Tobias Lamm

KIT Karlsruhe, Germany

The goal of this talk is to introduce the audience to the theory of diffusive stability in the context of the harmonic map flow. This theory is useful when studying stability results for parabolic equations and we will illustrate its use for geometric equations such as the harmonic map flow. Additionally, we use this theory in order improve various uniqueness results for solutions with rough initial data.